Practice Exam Paper

1 a $mv\frac{\mathrm{d}v}{\mathrm{d}x} = -4x$

$m = \frac{3}{2}$

$\frac{3}{2}v\frac{\mathrm{d}v}{\mathrm{d}x} = -4x$

$v\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{8}{3}x$

$\frac{1}{2}v^2 = -\frac{4}{3}x^2 + c$

$v^2 = -\frac{8}{3}x^2 + c$

When $v = 8$, $x = 0$

$(8)^2 = -\frac{8}{3}(0)^2 + c$

$c = 64$

Therefore

$v^2 = -\frac{8}{3}x^2 + 64$

b When the particle stops $v = 0$, therefore:

$(0)^2 = -\frac{8}{3}x^2 + 64$

$\frac{8}{3}x^2 = 64$

$x^2 = 24$

$x = 2\sqrt{6}$ m

2 a $S = \pi\int_0^{\frac{\pi}{4}} y^2\,\mathrm{d}x$

$= \pi\int_0^{\frac{\pi}{4}} \cos^2 x\,\mathrm{d}x$

$= \frac{\pi}{2}\int_0^{\frac{\pi}{4}} (1+\cos 2x)\,\mathrm{d}x$

$= \frac{\pi}{2}\left[x + \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}$

$= \frac{\pi}{2}\left[\frac{\pi}{4} + \frac{1}{2}(1)\right]$

$= \frac{\pi^2}{8} + \frac{\pi}{4}$

$= \frac{\pi}{8}(\pi + 2)$ as required.

b $\bar{x} = \dfrac{\int_0^{\frac{\pi}{4}} xy^2 \, dx}{\int_0^{\frac{\pi}{4}} y^2 \, dx}$

From part **a**:

$$\int_0^{\frac{\pi}{4}} y^2 \, dx = \frac{\pi}{8}(\pi + 2)$$

$$\int_0^{\frac{\pi}{4}} xy^2 \, dx = \int_0^{\frac{\pi}{4}} x\cos^2 x \, dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} x(1+\cos 2x) \, dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} x \, dx + \frac{1}{2}\int_0^{\frac{\pi}{4}} x\cos 2x \, dx$$

$$= \frac{1}{4}\left[x^2\right]_0^{\frac{\pi}{4}} + \frac{1}{2}\left[\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{4}\left[x^2\right]_0^{\frac{\pi}{4}} + \frac{1}{4}\left[x\sin 2x\right]_0^{\frac{\pi}{4}} + \frac{1}{8}\left[\cos 2x\right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{4}\left[\frac{\pi^2}{16}\right] + \frac{1}{4}\left[\frac{\pi}{4}(1)\right] - \frac{1}{8}[1]$$

$$= \frac{\pi^2}{64} + \frac{\pi}{16} - \frac{1}{8}$$

$$= \frac{\pi^2 + 4\pi - 8}{64}$$

$$\bar{x} = \frac{\frac{\pi^2 + 4\pi - 8}{64}}{\frac{1}{8}(\pi + 2)}$$

$$= \frac{\pi^2 + 4\pi - 8}{8(\pi + 2)}$$

3 **a** $F = \frac{GmM}{x^2}$

$G = \frac{gR^2}{M}$

$F = \frac{gR^2}{M} \times \frac{mM}{x^2}$

$= \frac{mgR^2}{x^2}$

b $mv\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{mgR^2}{x^2}$

$v\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{gR^2}{x^2}$

$\int_{2U}^{0} v\,\mathrm{d}v = -gR^2 \int_{\frac{3R}{2}}^{x} \frac{1}{x^2}\,\mathrm{d}x$

$\int_{2U}^{0} v\,\mathrm{d}v = -gR^2 \int_{\frac{3R}{2}}^{x} \frac{1}{x^2}\,\mathrm{d}x$

$\frac{1}{2}\left[v^2\right]_{2U}^{0} = -gR^2\left[-\frac{1}{x}\right]_{\frac{3R}{2}}^{x}$

$-\frac{1}{2}(2U)^2 = gR^2\left(\frac{1}{x} - \frac{2}{3R}\right)$

$-2U^2 = gR^2\left(\frac{3R-2x}{3xR}\right)$

$-6U^2xR = 3gR^3 - 2gR^2x$

$-6U^2xR + 2gR^2x = 3gR^3$

$x = \frac{3gR^2}{2\left(gR - 3U^2\right)}$

This is from the centre of the Earth. We need from the surface, so using $h = x - R$ gives:

$h = \frac{3gR^2}{2\left(gR-3U^2\right)} - R$

$h = \frac{3gR^2 - 2R\left(gR-3U^2\right)}{2\left(gR-3U^2\right)}$

$= \frac{3gR^2 - 2gR^2 + 6RU^2}{2\left(gR-3U^2\right)}$

$= \frac{gR^2 + 6RU^2}{2\left(gR-3U^2\right)}$

$= \frac{R\left(gR+6U^2\right)}{2\left(gR-3U^2\right)}$ as required.

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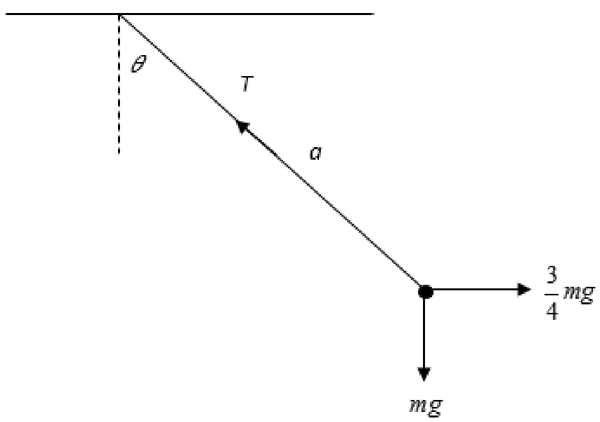

Res(←) $T\cos\theta = \frac{3}{4}mg$

Res(↑) $T\sin\theta = mg$

$$\frac{T\sin\theta}{T\cos\theta} = \frac{mg}{\frac{3}{4}mg}$$

$$\tan\theta = \frac{4}{3} \Rightarrow \sin\theta = \frac{4}{5} \text{ and } \cos\theta = \frac{3}{5}$$

$$\frac{4}{5}T = mg$$

$$T = \frac{5}{4}mg$$

By Hookes law:

$$T = \frac{\lambda x}{l}$$

$$x = \frac{Tl}{\lambda}$$

Substituting gives:

$$x = \frac{\frac{5}{4}mga}{2mg}$$

$$= \frac{5}{8}a$$

5 (to come)

6 a Let the P have zero potential energy.

Initial KE = $\frac{1}{2}\times 2m\times u^2 = mu^2$

Initial PE = $-2mg\times\frac{3a}{2} = -3mga$

Generally KE = $\frac{1}{2}\times 2m\times v^2 = mv^2$

and PE = $-2mg\times\frac{3a}{2}\cos\theta = -3mga\cos\theta$

Using conservation of energy:

$mu^2 - 3mga = mv^2 - 3mga\cos\theta$

$v^2 = u^2 - 3ga + 3ga\cos\theta$

The force acting towards the centre is:

$T - 2mg\cos\theta = \frac{2mv^2}{\frac{3}{2}a}$

$T = \frac{4m\left(u^2 - 3ga + 3ga\cos\theta\right)}{3a} + 2mg\cos\theta$

$= \frac{4mu^2}{3a} - 4mg + 6mg\cos\theta$

At the top, $T \geq 0$ and $\theta = 180°$

$\frac{4mu^2}{3a} - 4mg - 6mg \geq 0$

$\frac{4mu^2}{3a} \geq 10mg$

$u^2 \geq \frac{15}{2}ga$

$u \geq \sqrt{\frac{15}{2}ga}$

b i When $u = \sqrt{5ga}$ and $v = \sqrt{ga}$

Substituting into $v^2 = u^2 - 3ga + 3ga\cos\theta$ gives:

$ga = 5ga - 3ga + 3ga\cos\theta$

$3\cos\theta = -1$

$\cos\theta = -\frac{1}{3}$

$\theta = 109.5°$ (4 s.f.)

ii height $= \frac{3}{2}a\left(1-\cos\theta\right)$

$= \frac{3}{2}a\left(1-\left(-\frac{1}{3}\right)\right)$

$= 2a$

7 a Res(↑) $T = 2g = 19.6$

By Hookes law:

$$T = \frac{\lambda x}{l} = \frac{49x}{1.2}$$

$$\frac{49x}{1.2} = 19.6$$

$x = 0.48$ m

b Using $F = ma$

$$2g - T = 2\ddot{x}$$

$$2g - \frac{49}{1.2}(0.48 + x) = 2\ddot{x}$$

$$2\ddot{x} = -\frac{245}{6}x$$

$$\ddot{x} = -\frac{245}{12}x \text{ therefore SHM}$$

$$\omega = \sqrt{\frac{245}{12}} \text{ and } T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\sqrt{\frac{245}{12}}}$$

$$T^2 = \frac{4\pi^2}{\frac{245}{12}}$$

$$= \frac{48\pi^2}{245}$$

$$T = \frac{4}{7}\pi\sqrt{\frac{3}{5}} \text{ s}$$

c $v^2 = \omega^2(a^2 - x^2)$

At E, $v = \frac{2}{35}$ and $x = 0$

Substituting gives:

$$\frac{4}{1225} = \frac{245}{12}a^2$$

$a = 0.0126$ m

Therefore the particle moves 0.0126 m before coming to rest.

d $x = a\cos(\omega t)$

$x = -0.01$, $a = 0.0126$, $\omega = \sqrt{\frac{245}{12}}$

Rearrange to find

$$t = \frac{\arccos(x/a)}{\omega} = 0.55 \text{ seconds}$$